



In the 3×2 multiplication table below, the numbers 2, 3, 5, 7 and 11 are used to replace the variables a , b , c , d , and e .

\times	a	b	c
d			
e			

The six products are then found, and the sum of the products is determined. What is the maximum possible sum of the six products?



Solution: 196.

The solution to this problem can be found by trial-and-error. There are ${}_5C_2 = 10$ arrangements of the numbers. Once those arrangements are determined, the work of computing the products and resulting sum can be handled by a spreadsheet or calculator.

The ten possible arrangements are shown below:

	5	7	11
2	10	14	22
3	15	21	33

	3	7	11
2	6	14	22
5	15	35	55

	3	5	11
2	6	10	22
7	21	35	77

	3	5	7
2	6	10	14
11	33	55	77

	2	7	11
3	6	21	33
5	10	35	55

	2	5	11
3	6	15	33
7	14	35	77

	2	5	7
3	6	15	21
11	22	55	77

	2	3	11
5	10	15	55
7	14	21	77

	2	3	7
5	10	15	35
11	22	33	77

	2	3	5
7	14	21	35
11	22	33	55

The third arrangement in the center row gives a sum of $6 + 15 + 21 + 22 + 55 + 77 = 196$.

More interesting than the specific solution, however, is the solution to the general case. The general solution can be seen visually if the five numbers represent portions of the side length of a rectangle; then, the products within the multiplication table represent areas. The area of the entire figure (which will be equal to the sum of the six products) is maximized when the sums of the numbers along the top and side are as close to equal as possible. This makes sense, since a square is the geometric figure of maximum area with a fixed perimeter.

